NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1425

ON THE CALCULATION OF SHALLOW SHELLS

By S. A. Ambartsumyan

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1. We shall consider a sufficiently thin shallow shell of nonzero Gaussian curvature. In this case, neglecting certain small magnitudes, the problem, as shown by V. Z. Vlasov (ref. 1), can be reduced to a system of symmetrically constructed differential equations as follows:

$$\frac{1}{E\delta}\nabla_e^2\nabla_e^2\Phi - (H\nabla_e^2 - L\nabla_h^2)w = 0$$

$$-(H\nabla_e^2 - L\nabla_h^2)\Phi - \frac{E\delta^3}{12(1-\nu^2)}\nabla_e^2\nabla_h^2 w + Z = 0 \qquad (1.1)$$

These equations were constructed by the mixed method through the introduction of only two functions, namely, the stress function Φ, and the displacement function w.

The forces T_1, T_2, and S are expressed through Φ as follows:

$$T_1 = \frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{1}{B}\frac{\partial\Phi}{\partial\beta}\right) + \frac{1}{A^2B}\frac{\partial B}{\partial\alpha}\frac{\partial\Phi}{\partial\alpha}, \qquad T_2 = \frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{A}\frac{\partial\Phi}{\partial\alpha}\right) + \frac{1}{B^2A}\frac{\partial A}{\partial\beta}\frac{\partial\Phi}{\partial\beta}$$

$$S_1 = -S_2 = -\frac{1}{AB}\left(\frac{\partial^2\Phi}{\partial\alpha\partial\beta} - \frac{1}{A}\frac{\partial B}{\partial\alpha}\frac{\partial\Phi}{\partial\beta} - \frac{1}{B}\frac{\partial A}{\partial\beta}\frac{\partial\Phi}{\partial\alpha}\right) \qquad (1.2)$$

For the transverse forces N_1 and N_2, we have the formulas

$$N_1 = -\frac{E\delta^3}{12(1-\nu^2)}\frac{1}{A}\frac{\partial}{\partial\alpha}\nabla_e^2 w, \qquad N_2 = -\frac{E\delta^3}{12(1-\nu^2)}\frac{1}{B}\frac{\partial}{\partial\beta}\nabla_e^2 w \qquad (1.3)$$

*"K raschetu pologikh obolochek." Prik. Mat. i Mekh., vol. XI, 1947, pp. 527-532.

In these formulas, δ denotes the constant thickness of the shell, ν the Poisson coefficient, E the modulus of elasticity, $A = A(\alpha,\beta)$ and $B = B(\alpha,\beta)$ the coefficients of the first quadratic form of Gauss, $k_1 = k_1(\alpha,\beta)$ and $k_2 = k_2(\alpha,\beta)$ the principal curvatures of the coordinate surface in the orthogonal coordinate curves, β = constant, and α = constant. Further, the differential operators of the second order of the elliptical and hyperbolical type are defined as follows:

$$\nabla_e^2 = \frac{1}{AB}\left[\frac{\partial}{\partial\alpha}\left(\frac{B}{A}\frac{\partial}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{A}{B}\frac{\partial}{\partial\beta}\right)\right]$$

$$\nabla_h^2 = \frac{1}{AB}\left[B^2\frac{\partial}{\partial\alpha}\left(\frac{1}{AB}\frac{\partial}{\partial\alpha}\right) - A^2\frac{\partial}{\partial\beta}\left(\frac{1}{AB}\frac{\partial}{\partial\beta}\right)\right] \tag{1.4}$$

The mixed operator $H\nabla_e^2 - L\nabla_h^2$, in which $H = 1/2\ (k_1 + k_2)$ and $L = 1/2\ (k_1 - k_2)$, is defined by the formula

$$\nabla_k^2 = \left(H\nabla_h^2 - L\nabla_h^2\right) = \frac{1}{AB}\left[\frac{\partial}{\partial\alpha}\left(\frac{B}{A}k_2\frac{\partial}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{A}{B}k_1\frac{\partial}{\partial\beta}\right)\right] \tag{1.5}$$

2. We present equations (1.1) in a somewhat different form, so that

$$\frac{1}{E\delta}\nabla_e^4\ \Phi - \nabla_k^2\ w = 0, \qquad -\nabla_k^2\ \Phi - D\nabla_e^4\ w + Z = 0, \qquad \left(D = \frac{E\delta^3}{12(1 - \nu^2)}\right) \tag{2.1}$$

As shown by B. G. Galerkin (ref. 2), we note that the first equation of the system (2.1) is satisfied by introducing a certain displacement function $\varphi(\alpha,\beta)$, through which the required unknowns of the system (2.1) are expressed as

$$w = \nabla_e^4\varphi, \qquad \Phi = E\delta\nabla_k^2\varphi \tag{2.2}$$

The second equation of equations (2.1), then assumes the form

$$\nabla_e^8\varphi + \frac{12(1 - \nu^2)}{\delta^2}\nabla_k^4\varphi = \frac{Z}{D} \tag{2.3}$$

Considering equation (2.2), we give the formulas expressing the computational magnitudes in terms of the displacement function as follows:

$$T_1 = \frac{E\delta}{B}\left[\frac{\partial}{\partial\beta}\left(\frac{1}{B}\frac{\partial}{\partial\beta}\right) + \frac{1}{A^2}\frac{\partial B}{\partial\alpha}\frac{\partial}{\partial\alpha}\right]\nabla_k^2\varphi, \quad T_2 = \frac{E\delta}{A}\left[\frac{\partial}{\partial\alpha}\left(\frac{1}{A}\frac{\partial}{\partial\alpha}\right) + \frac{1}{B^2}\frac{\partial A}{\partial\beta}\frac{\partial}{\partial\beta}\right]\nabla_k^2\varphi$$

$$S_1 = -S_2 = -\frac{E\delta}{AB}\left[\frac{\partial^2}{\partial\alpha\partial\beta} - \frac{1}{A}\frac{\partial B}{\partial\alpha}\frac{\partial}{\partial\beta} - \frac{1}{B}\frac{\partial A}{\partial\beta}\frac{\partial}{\partial\alpha}\right]\nabla_k^2\varphi \tag{2.4}$$

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$$G_1 = D\left[\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{A}\frac{\partial}{\partial\alpha}\right) + \frac{1}{AB^2}\frac{\partial A}{\partial\beta}\frac{\partial}{\partial\beta} + \frac{\nu}{B}\frac{\partial}{\partial\beta}\left(\frac{1}{B}\frac{\partial}{\partial\beta}\right) + \frac{\nu}{A^2B}\frac{\partial B}{\partial\alpha}\frac{\partial}{\partial\alpha}\right]\nabla_e^4\varphi$$

$$G_2 = D\left[\frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{1}{B}\frac{\partial}{\partial\beta}\right) + \frac{1}{A^2B}\frac{\partial B}{\partial\alpha}\frac{\partial}{\partial\alpha} + \frac{\nu}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{A}\frac{\partial}{\partial\alpha}\right) + \frac{\nu}{AB^2}\frac{\partial A}{\partial\beta}\frac{\partial}{\partial\beta}\right]\nabla_e^4\varphi \qquad (2.5)$$

$$H_1 = -H_2 = -\frac{D}{AB}(1 - \nu)\left[\frac{\partial^2}{\partial\alpha\partial\beta} - \frac{1}{A}\frac{\partial B}{\partial\alpha}\frac{\partial}{\partial\beta} - \frac{1}{B}\frac{\partial A}{\partial\beta}\frac{\partial}{\partial\alpha}\right]\nabla_e^4\varphi$$

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$$N_1 = -\frac{D}{A}\frac{\partial}{\partial\alpha}\nabla_e^6\varphi, \qquad N_2 = -\frac{D}{B}\frac{\partial}{\partial\beta}\nabla_e^6\varphi \qquad (2.6)$$

Thus, the problem of computing shallow thin shells with arbitrary normally applied loads reduces to finding the displacement function $\varphi = \varphi(\alpha,\beta)$, which is determined by the differential equation (2.3).

3. Investigations by Y. N. Rabotnov (ref. 3) and A. L. Goldenveizer (ref. 4), show that the coefficients of the first quadratic form A and B, for a certain part of an arbitrary shell, behave almost like constants in differentiation. Hence, in the differentiation of products of the form Aw (or kw), the derivative of A may be neglected, and we can set $d(Aw) \approx Adw$.

If it is assumed that α and β are absolute coordinates, then, on the basis of the previous discussion, we may set $A \approx B \approx 1$; we then obtain

$$\nabla_e^2 = \nabla^2 = \frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\beta^2} \qquad \nabla_k^2 = k_2\frac{\partial^2}{\partial\alpha^2} + k_1\frac{\partial^2}{\partial\beta^2} \qquad (3.1)$$

Equation (2.3) then takes the form

$$\nabla^8\varphi + \frac{12(1 - \nu^2)}{\delta^2}\nabla_k{}^4\varphi = \frac{Z}{D} \qquad (3.2)$$

Now equations (2.4), (2.5), (2.6), and (fig. 1), are determined by the formulas

$$T_1 = E\delta\frac{\partial^2}{\partial\beta^2}\nabla_k^2\varphi, \qquad S = -E\delta\frac{\partial^2}{\partial\alpha\partial\beta}\nabla_k^2\varphi, \qquad G_1 = D\left[\frac{\partial^2}{\partial\alpha^2} + \nu\frac{\partial^2}{\partial\beta^2}\right]\nabla^4\varphi$$

$$T_2 = E\delta\frac{\partial^2}{\partial\alpha^2}\nabla_k^2\varphi, \qquad H = -D(1 - \nu)\frac{\partial^2}{\partial\alpha\partial\beta}\nabla^4\varphi, \quad G_2 = D\left[\frac{\partial^2}{\partial\beta^2} + \nu\frac{\partial^2}{\partial\alpha^2}\right]\nabla^4\varphi \qquad (3.3)$$

$$N_1 = -D\frac{\partial}{\partial\alpha}\nabla^6\varphi, \qquad Q_1 = N_1 + \frac{\partial H}{\partial\beta} = -D\left[\frac{\partial}{\partial\alpha}\nabla^2 + (1-\nu)\frac{\partial^3}{\partial\alpha\partial\beta^2}\right]\nabla^4\varphi$$

$$N_2 = -D\frac{\partial}{\partial\beta}\nabla^6\varphi, \qquad Q_2 = N_2 + \frac{\partial H}{\partial\alpha} = -D\left[\frac{\partial}{\partial\beta}\nabla^2 + (1-\nu)\frac{\partial^3}{\partial\alpha^2\partial\beta}\right]\nabla^4\varphi \tag{3.4}$$

4. As an example, let us consider a shallow shell, rectangular in the plane, freely supported on its contour, and subjected to a normal load. Let a and b denote the dimensions of the shell in the directions α and β. 4225

The boundary conditions of the problem are

$$\begin{aligned} &w = 0, \quad u = 0, \quad T_1 = 0, \quad G_1 = 0 \quad \text{for} \quad \alpha = 0, \quad \alpha = a \\ &w = 0, \quad v = 0, \quad T_2 = 0, \quad G_2 = 0 \quad \text{for} \quad \beta = 0, \quad \beta = b \end{aligned} \tag{4.1}$$

These boundary conditions are satisfied by a solution of the form

$$\varphi = \sum_m^\infty \sum_n^\infty A_{mn} \sin\frac{m\pi\alpha}{a} \sin\frac{n\pi\beta}{b} \tag{4.2}$$

Expanding the external normally applied load into a double trigonometric series, and substituting the results obtained in equation (3.2), yield

$$A_{mn} = \frac{4a^8}{D\pi^8 ab\Delta'_{mn}}\int_0^a\int_0^b Z \sin\frac{m\pi\alpha}{a}\sin\frac{n\pi\beta}{b}\,d\alpha\,d\beta \tag{4.3}$$

where m and n are odd positive numbers,

$$\Delta'_{mn} = \left[(m^2 + \lambda^2 n^2)^4 + C\lambda_{mn}^2\right] \qquad (\alpha \geq b, \ \lambda = \alpha/b) \tag{4.4}$$

$$\lambda_{mn} = (Km^2 + \lambda^2 n^2), \qquad K = \frac{R_1}{R_2} = \frac{k_2}{k_1}, \qquad C = \frac{12(1-\nu^2)a^4}{\pi^4 R_1^2 \delta^2} \tag{4.5}$$

5. Let us consider the loading of the shell by a concentrated force P, applied at an arbitrary point (η,ξ). According to equation (4.3), we have

$$A_{mn} = -\frac{4Pa^8}{D\pi^8 ab\Delta'_{mn}}\sin\frac{m\pi\eta}{a}\sin\frac{n\pi\xi}{b} \tag{5.1}$$

Hence, from equations (4.2) and (3.3), we have

$$\varphi = -\frac{4Pa^2}{D\pi^8 ab}\sum_m\sum_n\frac{1}{\Delta'_{mn}}\; S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi} \tag{5.2}$$

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$$T_1 = -\frac{4P\lambda^3 R_1}{a^2}\, c\sum_m\sum_n\frac{n^2\lambda_{mn}}{\Delta'_{mn}}\; S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}$$

$$T_2 = -\frac{4P\lambda R_1}{a^2}\, c\sum_m\sum_n\frac{m^2\lambda_{mn}}{\Delta'_{mn}}\; S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}$$

$$S = -\frac{4P\lambda^2 R_1}{a^2}\, c\sum_m\sum_n\frac{mn\lambda_{mn}}{\Delta'_{mn}}\; C_{m\alpha}C_{n\beta}S_{m\eta}S_{n\xi} \tag{5.3}$$

In the present and succeeding discussion, there is introduced for brevity, the notations .

$$\sin\frac{m\pi\alpha}{a} = S_{m\alpha}, \quad \sin\frac{n\pi\beta}{b} = S_{n\beta}, \quad \sin\frac{m\pi\eta}{a} = S_{m\eta}, \quad \sin\frac{n\pi\xi}{b} = S_{n\xi}$$

$$\cos\frac{m\pi\alpha}{a} = C_{m\alpha}, \quad \cos\frac{n\pi\beta}{b} = C_{n\beta}, \quad \cos\frac{m\pi\eta}{a} = C_{m\eta}, \quad \cos\frac{n\pi\xi}{b} = C_{n\xi} \tag{5.4}$$

Further, from equation (2.2), we obtain

$$w = -\frac{4Pa^4}{D\pi^4 ab}\sum_m\sum_n\frac{(m^2+\lambda^2n^2)}{\Delta'_{mn}}\; S_{m\alpha}S_{n\beta}S_{m\eta}S_n \tag{5.5}$$

Making use of the identity

$$\sum_m\sum_n\frac{(m^2+\lambda^2n^2)}{\Delta'_{mn}} = \sum_m\sum_n\frac{1}{(m^2+\lambda^2n^2)^2} - c\sum_m\sum_n\frac{\lambda^2_{mn}}{\Delta''_{mn}} \tag{5.6}$$

where

$$\Delta''_{mn} = (m^2+\lambda^2n^2)^2\,\Delta'_{mn} \tag{5.7}$$

we obtain from equation (5.5),

$$w = -\frac{4Pa^4}{D\pi^4 ab}\sum_m\sum_n \frac{S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}}{(m^2 + \lambda^2 n^2)^2} + \frac{4Pa^4}{D\pi^4 ab} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}$$

The first term of this formula is the expression for the deflection of a rectangular plate, freely supported on its contour at the sides a and b under a concentrated load P, and applied at an arbitrary point (η,ξ). Denoting the deflection of the plate by w^*, we obtain

$$w = \nabla^4\varphi = -\left[w^* - \frac{4Pa^4}{D\pi^4 ab} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}\right] \qquad (5.8)$$

The possibility of separating the deflection of a plate from the expression for the displacement w of a circular cylindrical shell was first shown by T. T. Khachatryan, in an unpublished doctoral dissertation.

Further, making use of the new expression (5.8), we obtain

$$G_1 = M_1^* - \frac{4P\lambda}{\pi^2} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} (m^2 + \nu\lambda^2 n^2) S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}$$

$$G_2 = M_2^* - \frac{4P\lambda}{\pi^2} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} (\lambda^2 n^2 + \nu m^2) S_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}$$

$$H = H^* - \frac{4P\lambda^2}{\pi^2} (1 - \nu) C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} mn\, C_{m\alpha}C_{n\beta}S_{m\eta}S_{n\xi}$$

$$N_1 = N_1^* - \frac{4P\lambda}{\pi a} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} m (m^2 + \lambda^2 n^2) C_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi} \qquad (5.9)$$

$$N_2 = N_2^* - \frac{4P\lambda}{\pi b} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} n (m^2 + \lambda^2 n^2) S_{m\alpha}C_{n\beta}S_{m\eta}S_{n\xi}$$

$$Q_1 = Q_1^* - \frac{4P\lambda}{\pi a} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} m (m^2 + \nu\lambda^2 n^2) C_{m\alpha}S_{n\beta}S_{m\eta}S_{n\xi}$$

$$Q_2 = Q_2^* - \frac{4P\lambda}{\pi b} C \sum_m\sum_n \frac{\lambda_{mn}^2}{\Delta''_{mn}} n (\lambda^2 n^2 + \nu m^2) S_{m\alpha}C_{n\beta}S_{m\eta}S_{n\xi}$$

In the preceding formulas, the magnitudes denoted with asterisks correspond to the bending of a rectangular plate (a×b), coinciding with the contour of the shell. For computing these magnitudes, the well-known tables of B. G. Galerkin (ref. 5) may be used.

The remaining magnitudes in formulas (5.9) represent the effect of the additional internal forces that arise as a result of the curvature of the shell (the curvatures k_1 and k_2 enter in equation (2.1) in the composition of the secondary differential operator of the mixed type, and play the part of coefficient of elastic base, with respect to the flat plate).

By an analogous method, the formulas may be obtained for a dense, uniformly distributed load q = constant, over the entire surface of the shell, and are shown as follows:

$$\varphi = -\frac{16qa^8}{D\pi^{10}}\sum_m\sum_n\frac{S_{m\alpha}S_{n\beta}}{mn\Delta'_{mn}}$$

$$T_1 = -\frac{16q\lambda^2R_1}{\pi^2}C\sum_m\sum_n\frac{n\lambda_{mn}}{m\Delta'_{mn}}S_{m\alpha}S_{n\beta}$$

$$T_2 = -\frac{16qR_1}{\pi^2}C\sum_m\sum_n\frac{m\lambda_{mn}}{n\Delta'_{mn}}S_{m\alpha}S_{n\beta}$$

$$S = -\frac{16q\lambda R_1}{\pi^2}C\sum_m\sum_n\frac{\lambda_{mn}}{\Delta'_{mn}}C_{m\alpha}C_{n\beta}$$

$$w = -\left[w^* - \frac{16qa^4}{D\pi^6}C\sum_m\sum_n\frac{\lambda_{mn}}{mn\Delta''_{mn}}S_{m\alpha}S_{n\beta}\right]$$

$$G_1 = M_1^* - \frac{16qa^2}{\pi^4}C\sum_m\sum_n\frac{\lambda_{mn}^2(m^2 + \nu\lambda^2n^2)}{mn\Delta''_{mn}}S_{m\alpha}S_{n\beta}$$

$$G_2 = M_2^* - \frac{16qa^2}{\pi^4}C\sum_m\sum_n\frac{\lambda_{mn}^2(\lambda^2n^2 + \nu m^2)}{mn\Delta''_{mn}}S_{m\alpha}S_{n\beta}$$

$$H = H^* - \frac{16q\lambda a^2}{\pi^2}(1 - \nu)C\sum_m\sum_n\frac{\lambda_{mn}^2}{\Delta''_{mn}}C_{m\alpha}C_{n\beta}$$

$$N_1 = N_1^* - \frac{16qa}{\pi^3}C\sum_m\sum_n\frac{\lambda_{mn}^2(m^2 + \lambda^2n^2)}{n\Delta''_{mn}}C_{m\alpha}S_{n\beta}$$

$$N_2 = N_2^* - \frac{16qa\lambda}{\pi^3}C\sum_m\sum_n\frac{\lambda_{mn}^2(m^2 + \lambda^2n^2)}{m\Delta''_{mn}}S_{m\alpha}C_{n\beta}$$

6. In conclusion, a numerical example is given of the computation of a shell, rectangular in the plane, and freely supported over the contour for a uniformly distributed normal load.

We take $\lambda = a/b = 2$, the ratio $K = R_1/R_2 = 2$, the Poisson coefficient $\nu = 0.3$, the intensity of the load q, and Young's modulus E. According to equations (4.5), $C = 20$.

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For the deflection w, we obtain the formula

$$w = -\frac{qb^4}{E\delta^3}\left(w^* - \Delta w\right) \tag{6.1}$$

where w^* is a numerical coefficient taken from the previously mentioned tables (ref. 5), and Δw is an additional deflection depending on the curvature of the shell, so that

$$\Delta w = \frac{16\times 12\left(1 - \nu^2\right) C\lambda^4}{\pi^6} \sum_m \sum_n \frac{\lambda_{mn}^2}{mn\Delta''_{mn}} \sin\frac{m\pi\alpha}{a} \sin\frac{n\pi\beta}{b} \tag{6.2}$$

Restricting ourselves to two terms of the expansion in formula (6.2), the following results of the computations for several points (α,β) of the shell are given:

Points	$\left(\alpha = \frac{1}{4}a,\ \beta = \frac{1}{2}b\right)$	$\left(\alpha = \frac{1}{2}a,\ \beta = \frac{1}{2}b\right)$	$\left(\alpha = \frac{1}{2}a,\ \beta = \frac{1}{4}b\right)$
w^*	0.0843	0.1106	0.0760
Δw	.0449	.0608	.0431
$w^* - \Delta w$	.0394	.0498	.0329

For the bending moments G_1 and G_2, we obtain the formulas

$$G_1 = qb^2\left(m_1^* - \Delta m_1\right), \qquad G_2 = qb^2\left(m_2^* - \Delta m_2\right) \tag{6.3}$$

where m_1^* and m_2^* are coefficients taken from table III (ref. 5), and

$$\left.\begin{aligned}
\Delta m_1 &= \frac{16\lambda^2 C}{\pi^4} \sum_m \sum_n \frac{\lambda_{mn}^2\left(m^2 + \nu\lambda^2 n^2\right)}{mn\Delta''_{mn}} \sin\frac{m\pi\alpha}{a} \sin\frac{n\pi\beta}{b} \\
\Delta m_2 &= \frac{16\lambda^2 C}{\pi^4} \sum_m \sum_n \frac{\lambda_{mn}^2\left(\lambda^2 n^2 + \nu m^2\right)}{mn\Delta''_{mn}} \sin\frac{m\pi\alpha}{a} \sin\frac{n\pi\beta}{b}
\end{aligned}\right\} \tag{6.4}$$

Restricting ourselves to two terms of the expansion in formula (6.4), the following results of the computation for several points are given:

Points	$\left(\alpha = \frac{1}{4} a, \beta = \frac{1}{2} b\right)$	$\left(\alpha = \frac{1}{2} a, \beta = \frac{1}{2} b\right)$	$\left(\alpha = \frac{1}{2} a, \beta = \frac{1}{4} b\right)$
m_1^*	0.0446	0.0464	0.0304
Δm_1	.0241	.0276	.0195
$m_1^* - \Delta m_1$	.0205	.0188	.0109
m_2^*	.0807	.1017	.0773
Δm_2	.0444	.0583	.0412
$m_2^* - \Delta m_2$	.0363	.0434	.0361

For the forces T_1 and T_2, we have the formulas

$$T_1 = - qR_1\Delta t_1, \qquad T_2 = - qR_1\Delta t_2 \tag{6.5}$$

where

$$\Delta t_1 = \frac{16\lambda^2 C}{\pi^2} \sum_m \sum_n \frac{n\lambda_{mn}}{m\Delta'_{mn}} \sin \frac{m\pi\alpha}{a} \sin \frac{n\pi\beta}{b}$$

$$\Delta t_2 = \frac{16C}{\pi^2} \sum_m \sum_n \frac{m\lambda_{mn}}{n\Delta''_{mn}} \sin \frac{m\pi\alpha}{a} \sin \frac{n\pi\beta}{b}$$

Restricting ourselves to three terms of the expansion in these formulas, the results of the computations are:

Points	$\left(\alpha = \frac{1}{4} a, \beta = \frac{1}{2} b\right)$	$\left(\alpha = \frac{1}{2} a, \beta = \frac{1}{2} b\right)$	$\left(\alpha = \frac{1}{2} a, \beta = \frac{1}{4} b\right)$
Δt_1	0.4252	0.5569	0.3928
Δt_2	.1333	.1010	.0711

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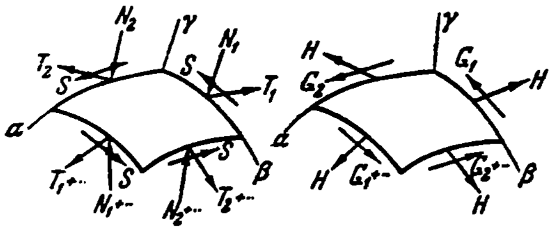
N_2
γ
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Figure 1.